

Copy Number 39

Support Test

SVCC Math Contest Calculus Exam

Time Allowed: 50 Minutes
40 Problems

Directions: For this test, solve each problem. Then indicate the best answer in the appropriate space on the answer sheet. Please stay in your seat until time is called.

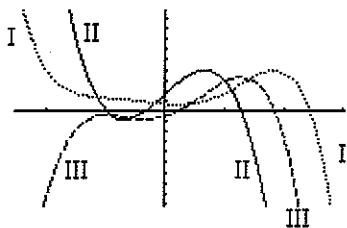
Please do not write on this test booklet.

SVCC Math Contest

Calculus Test

- If $f(2) = 3$, $g(2) = -3$, $f'(2) = 2$, and $g'(2) = 5$, find $h'(2)$ if $h = f \cdot g$.
A) 10
B) 4
C) 9
D) -6
E) -15
- Given two numbers whose sum is 400, find the maximum value of their product.
A) 30,000
B) 40,000
C) 35,000
D) 45,000
E) 50,000
- If $f(2) = 3$, $g(2) = -3$, $f'(2) = 2$, and $g'(2) = 5$, find $h'(2)$ if $h = f + g$.
A) 0
B) 7
C) -9
D) 10
E) Can't be calculated with this information.
- Given $x^2 + 3xy + y^2 = 5$, find $\frac{dy}{dx}$ at the point $(1, -4)$.
A) -2
B) 1
C) 0
D) 3
E) $(1, -4)$ is not a point on the graph of this equation.
- Given $f(x) = \sin(g(x))$ where $g(x)$ is a differentiable function. What is $f'(x)$?
A) $g(x) \cdot \cos(x)$
B) $-g'(x) \cdot \sin(x)$
C) $-g(x) \cdot \sin(x)$
D) $g'(x) \cdot \cos(x)$
E) $g'(x) \cdot \cos(g(x))$
- The volume of a cube is increasing at the rate of $2 \text{ cm}^3/\text{min}$. Find the rate of increase of the sides when the sides are 5 cm long.
A) $\sqrt[3]{5} \text{ cm/min}$
B) $\sqrt[3]{2} \text{ cm/min}$
C) $\frac{2}{75} \text{ cm/min}$
D) $\frac{2}{5} \text{ cm/min}$
E) $5\sqrt[3]{2} \text{ cm/min}$

7. Graphs I, II, and III below are the graphs of a function and its first and second derivatives. Which answer assigns the correct roles?

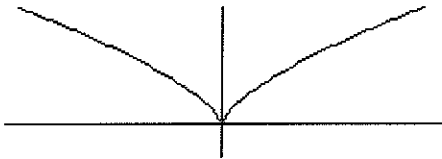


- A) I is the graph of the function, II the graph of its derivative, and III is the graph of its second derivative.
- B) II is the graph of the function, I is the graph of its derivative, and III is the graph of its second derivative.
- C) III is the graph of the function, II is the graph of its derivative, and I is the graph of its second derivative.
- D) I is the graph of the function, III is the graph of its derivative, and II is the graph of its second derivative.
- E) There is no way to tell which is which by simply looking at the graphs.
8. Find the area bounded by the graphs of $y = \sin(x)$, $y = \cos(x)$, $x = 0$, and $x = \pi/2$.
- A) 0 B) 1
- C) $2\sqrt{2} - 2$ D) $2\sqrt{2} + 2$
- E) 2
9. Given $f(x) = \begin{cases} -x^2 + 2 & -1 < x \leq 1 \\ |x| & 1 < x < 2 \\ 1 & x < -1 \text{ or } x \geq 2 \end{cases}$.
- On which of the intervals below is f continuous?
- A) $[-1, 2]$ B) $(-1, 2]$
- C) $[-1, 2)$ D) $(-1, 2)$
- E) None of the above
10. How many points of inflection does $f(x) = x^8 - x^2$ have?
- A) 0 B) 1
- C) 2 D) 3
- E) 4

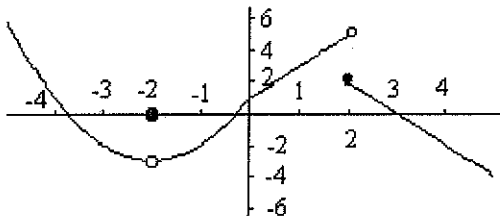
11. Given $h(x) = (f \circ g)(x)$ where f and g are both differentiable functions. Use the values in the table to find $h'(2)$.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
2	3	5	-2	2
3	-5	-3	-5	7
5	4	-1	-4	-6

- A) -4
 B) 10
 C) -10
 D) -8
 E) 8
12. The graph of $f'(x)$ is shown below. Pick the best description of the point $(0, f(0))$.

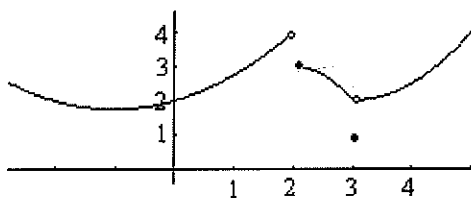


- A) $f(0)$ is a relative minimum.
 B) $f(0)$ is a relative maximum.
 C) $(0, f(0))$ is a critical point, but $f(0)$ is neither a maximum nor a minimum.
 D) There is nothing special about $(0, f(0))$.
 E) You can't tell about $(0, f(0))$ from the graph of $f'(x)$.
13. Use the graph of $f(x)$ below to choose the correct statement about $f(x)$.



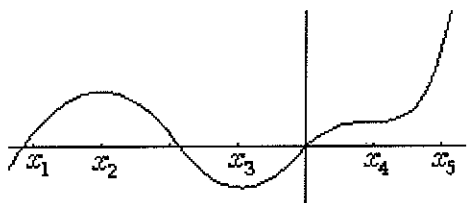
- A) f is continuous at $x = -2$.
 B) $\lim_{x \rightarrow -2} f(x) = -3$
 C) $\lim_{x \rightarrow -2^-} f(x) = 0$
 D) $\lim_{x \rightarrow -2^+} f(x) = 0$
 E) All the statements are true.

14. The graph of a function $y = f(x)$ is given below. Which of the following statement(s) is (are) true?



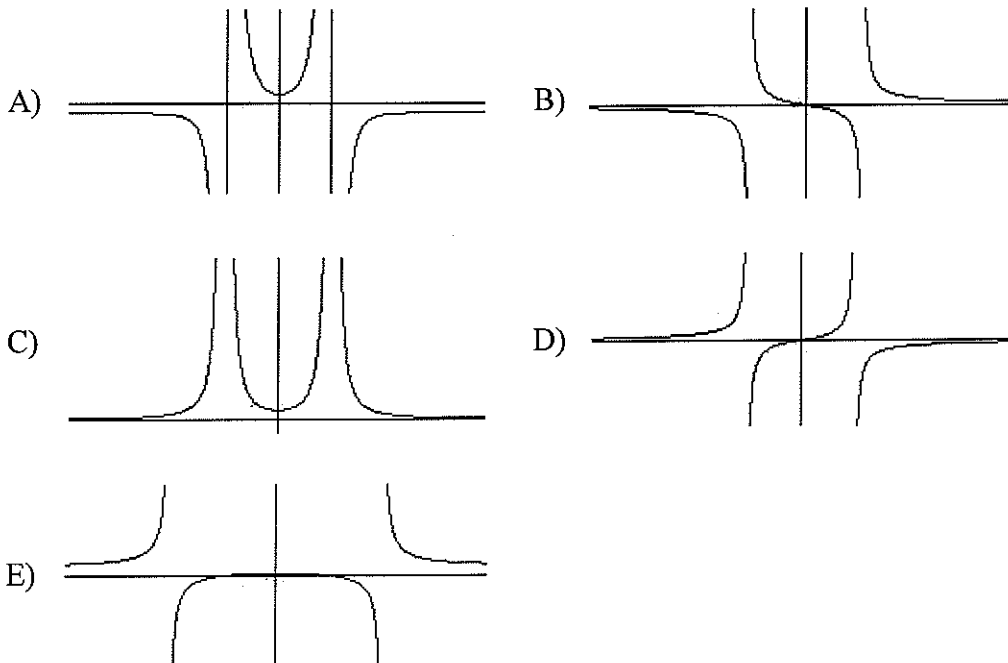
- A) $\lim_{x \rightarrow 2^-} f(x) = 4$ B) $\lim_{x \rightarrow 2^+} f(x) = 3$
 C) $\lim_{x \rightarrow 2} f(x) = \text{Does Not Exist}$ D) $\lim_{x \rightarrow 3} f(x) = 2$
 E) All of the above
15. Which of the following statements is false?
- A) If f is differentiable at $x = c$, then f is continuous at $x = c$.
 B) If f is continuous at $x = c$, then f is differentiable at $x = c$.
 C) If $\lim_{\Delta x \rightarrow 0} \frac{f(c + \Delta x) - f(c)}{\Delta x}$ exists, then f is differentiable at $x = c$.
 D) If $\lim_{t \rightarrow c^-} \frac{f(c) - f(t)}{c - t} = \lim_{t \rightarrow c^+} \frac{f(c) - f(t)}{c - t}$, then f is differentiable at $x = c$.
 E) None of the above

16. Given the graph of $f(x)$ below, which of the following statements is true?



- A) $f'(x) > 0$ on (x_1, x_2) B) $f'(x) < 0$ on (x_2, x_3)
 C) $f'(x) > 0$ on (x_3, x_4) D) $f'(x) > 0$ on (x_4, x_5)
 E) All of the above

17. Given: $\lim_{x \rightarrow \infty} f(x) = \lim_{x \rightarrow -\infty} f(x) = 0$, $\lim_{x \rightarrow -1^-} f(x) = \lim_{x \rightarrow 1^+} f(x) = -\infty$, and $\lim_{x \rightarrow -1^+} f(x) = \lim_{x \rightarrow 1^-} f(x) = \infty$, which of the following could be the graph of $f(x)$?



18. A plastic right circular cylinder with closed ends is to hold $V \text{ cm}^3$. Find the ratio of height to diameter that minimizes the amount of material required. Assume there is no waste in the construction process.

- A) 1 : 2
 B) π : 1
 C) 1 : 2π
 D) 2π : 1
 E) 1 : 1

19. What value of c will make $f(x) = \begin{cases} 2x + c & x < 1 \\ x^2 - cx & x \geq 1 \end{cases}$ continuous for all real values of x ?

- A) 1
 B) -1
 C) $-\frac{1}{2}$
 D) $\frac{1}{2}$
 E) 2

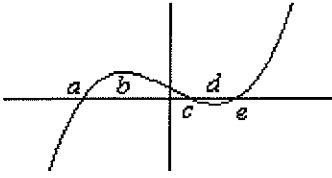
20. If $a_m \neq 0$ and $b_n \neq 0$, what condition(s) on n and m will guarantee that

$$r(x) = \frac{a_m x^m + a_{m-1} x^{m-1} + \cdots + a_1 x + a_0}{b_n x^n + b_{n-1} x^{n-1} + \cdots + b_1 x + b_0}$$

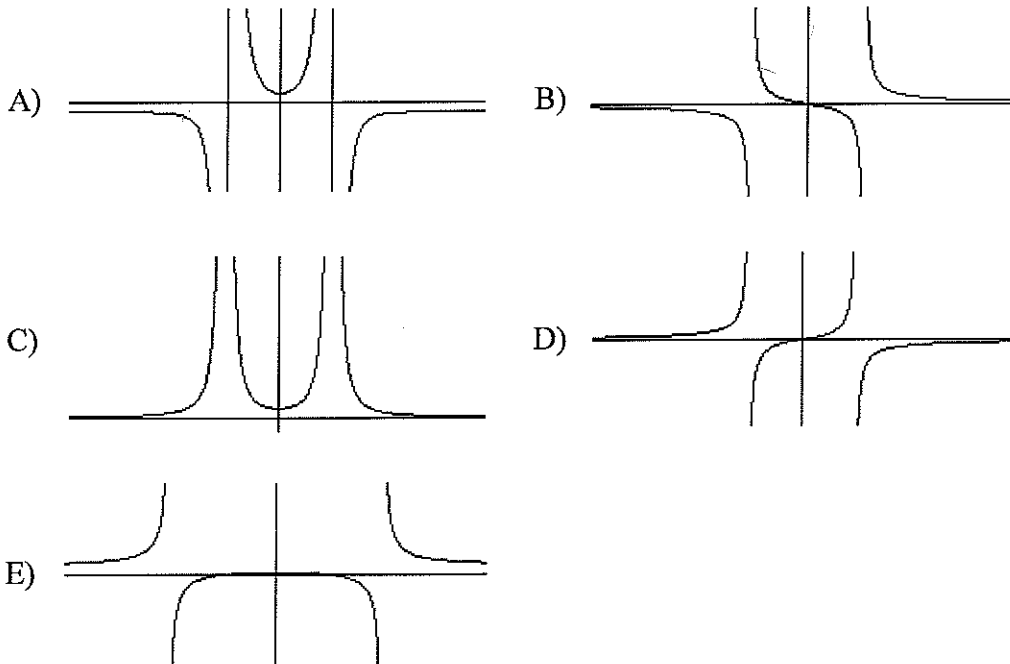
has a horizontal asymptote?

- A) $n < m$
 B) $n > m$
 C) $n = m$
 D) B and C
 E) A and B

21. The graph of $f'(x)$ is shown. Which answer best describes the behavior of f ?

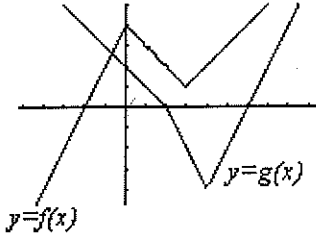


- A) f is increasing for $x < b$, decreasing on (b, d) , and increasing for $x > d$.
 B) f decreases for $x < a$ and for $c < x < e$ and increases otherwise.
 C) f increases for $a < x < b$ and for $x > e$ and decreases otherwise.
 D) f increases for all values of x except those which are less than a .
 E) There's no way to determine the behavior of f from this graph.
22. Given: $\lim_{x \rightarrow \infty} f(x) = \lim_{x \rightarrow -\infty} f(x) = 0$ and $\lim_{x \rightarrow -1} f(x) = \lim_{x \rightarrow 1} f(x) = \infty$, which of the following could be the graph of $f(x)$?



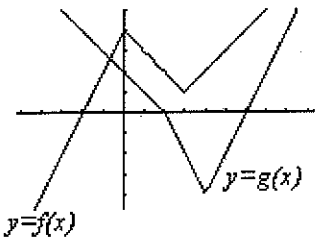
23. Given: $f(x) = 4x^4 + 4x$. Which of the following statements is true about f ?
- A) $x = -1$ is a critical number of f . B) $(0, 0)$ is an inflection point for f .
 C) f is concave down for $x < 0$. D) f is concave down for $x > 0$.
 E) None of the above

24. The graph of two functions f and g are shown below. If $p(x) = f(x) \cdot g(x)$, find $p'(1)$. Tick marks on both axes represent one unit.



- A) -4 B) 1
 C) -1 D) 4
 E) -2

25. The graph of two functions f and g are shown below. If $q(x) = \frac{f(x)}{g(x)}$, find $q'(5)$. Tick marks on both axes represent one unit.



- A) -4 B) 1
 C) -1 D) 4
 E) -2

The normal line to $f(x) = 2x^2 - 3x$ at the point $(2, 2)$.

- B) 5
 D) $\frac{12}{5}$

$\frac{5}{12}$

27. The edges of a cube are increasing at the rate of 0.13 cm/min. Find the rate at which the surface area of the cube is increasing when the edges are 2.7 cm long.
- A) 0.3510 cm²/min B) 4.212 cm²/min
 C) 2.7 cm²/min D) 0.13 cm²/min
 E) 19.683 cm²/min
28. $\frac{d}{dx} \left(\int_1^x \frac{e^t}{e^t + 1} dt \right) = ?$
- A) $\ln \left(\frac{e^x + 1}{e + 1} \right)$ B) 0
 C) $\ln(x) - \ln(1)$ D) $\frac{e^x}{e^x + 1}$
 E) Can't be determined
29. A stone tossed into a calm pond causes concentric circular ripples to spread from the point of entry. If the radius of the ripples increase at a constant rate of 3 inches/second, find the rate at which the area enclosed by one of the ripples when its diameter is 4 feet.
- A) 2π ft²/sec B) 16 ft²/sec
 C) π ft²/min D) 9 ft²/min
 E) π^2 ft²/min
30. Let $y = e^{g(x)}$ where $g(x)$ is a differentiable function. What is $\frac{dy}{dx}$?
- A) $e^{g(x)}$ B) $g(x) \cdot e^{g(x)-1}$
 C) $g'(x) \cdot e^{g(x)-1}$ D) $e^{g'(x)}$
 E) $g'(x) \cdot e^{g(x)}$
31. Find the slope of the secant line through $(x, f(x))$ and $(x + h, f(x + h))$ if $f(x) = x^3 - 4x^2$.
- A) $3x^2 - 8x$ B) $3x^2 - 8x + h$
 C) $3x^2 - 8x + 3hx + h^2 - 4h$ D) $3hx^2 - 8hx + 3h^2x + h^3 - 4h^2$
 E) The slope of the secant line is undefined.
32. Find the equation of the line through $(2, -2)$ which is parallel to the tangent line to $y = x^2$ at the point $(2, 2)$.
- A) $y = 2x - 6$ B) $y = 4x - 10$
 C) $y = -2x + 2$ D) $y = x - 4$
 E) $y = 3x - 8$

33. In order to approximate $\int_1^3 x^2 - x dx$, the interval $[1, 3]$ is partitioned into four subintervals of equal length, and the Riemann sum using the midpoints of the intervals as representative values of x is calculated. Its value is:

- A) $\frac{25}{4}$
- B) $\frac{13}{4}$
- C) $\frac{37}{8}$
- D) $\frac{19}{4}$

E) $\frac{14}{3}$

34. $\int \frac{x^2 + 1}{x} dx = ?$

A) $\frac{x^3 + x}{x^2} + C$

C) $2x + C$

E) $\frac{(x^2 + 1)^2}{2} \cdot \ln|x| + C$

B) $\frac{x^2}{2} + \ln|x| + C$

D) $\frac{2x^3 + 3x}{3x^2} + C$

35. $\int \sin(x) \cos(x) dx$

A) $\frac{\sin^2(x)}{2} + C$

C) $-\frac{\cos(2x)}{4} + C$

E) All of the above.

B) $-\frac{\cos^2(x)}{2} + C$

D) $-\frac{\cos(2x) + 1}{4} + C$

36. If $\int_0^4 f(x) dx = 5$ and $\int_0^4 g(x) dx = 2$ find $\int_0^4 f(x) \cdot g(x) dx$.

A) 10

C) $\frac{5}{2}$

E) Can't be determined.

B) 7

D) 3

37. If $\int_0^3 f(x) dx = 4$, $\int_3^6 f(x) dx = 4$, and $\int_2^6 f(x) dx = 5$, find $\int_0^2 f(x) dx$.

A) -3

C) 2

E) Can't be determined.

B) 3

D) 0

38. Evaluate: $\int_{-3}^3 |x| + |x + 1| dx$

A) 19

B) 16

C) 11

D) 0

E) 9

39. The expression $\lim_{n \rightarrow \infty} \sum_{i=1}^n \left(5 + \left(\frac{3i}{n} \right)^2 \right) \cdot \frac{3}{n}$ results in what definite integral?

A) $\int_1^5 x^2 dx$

B) $\int_1^2 5 + x^2 dx$

C) $\int_0^3 5 + x^2 dx$

D) $\int_0^n x^2 dx$

E) $\int_0^n 5 + x^2 dx$

40. Find the slope of the tangent line to the graph of $y = \sin(x)$ at the point $\left(\frac{\pi}{6}, \frac{1}{2}\right)$.

A) 0

B) $\sqrt{2}/2$

C) $1/2$

D) 1

E) $\sqrt{3}/2$

Student ID Number	0	1	2	3	4	5	6	7	8	9	0	1	2	3	4	5	6	7	8	9	0	1	2	3	4	5	6	7	8	9	0	1	2	3	4	5	6	7	8	9
	0	1	2	3	4	5	6	7	8	9	0	1	2	3	4	5	6	7	8	9	0	1	2	3	4	5	6	7	8	9	0	1	2	3	4	5	6	7	8	9

STUDENTS

LIST NO. 2 PENCIL ONLY

- EXAMPLE: \ominus \ominus \ominus \ominus \ominus \ominus
- MAKE DARK MARKS
- ERASE COMPLETELY TO CHANGE
- MAKE NO STRAY MARKS

INSTRUCTORS

KEY MARKING INSTRUCTIONS

KEY: Must mark this box on key sheet.
 VERIFY: Prints correct response next to marked answer. If Verify is not marked, a dash will print next to incorrect answers.
 RESCORE: Rescores a previously scored test. Automatically prints correct responses.

Student ID	(T)	(F)	(T)	(F)
1	\ominus A	\ominus B	\ominus C	\ominus D
2	\ominus A	\ominus B	\ominus C	\ominus D
3	\ominus A	\ominus B	\ominus C	\ominus D
4	\ominus A	\ominus B	\ominus C	\ominus D
5	\ominus A	\ominus B	\ominus C	\ominus D
6	\ominus A	\ominus B	\ominus C	\ominus D
7	\ominus A	\ominus B	\ominus C	\ominus D
8	\ominus A	\ominus B	\ominus C	\ominus D
9	\ominus A	\ominus B	\ominus C	\ominus D
10	\ominus A	\ominus B	\ominus C	\ominus D
11	\ominus A	\ominus B	\ominus C	\ominus D
12	\ominus A	\ominus B	\ominus C	\ominus D
13	\ominus A	\ominus B	\ominus C	\ominus D
14	\ominus A	\ominus B	\ominus C	\ominus D
15	\ominus A	\ominus B	\ominus C	\ominus D
16	\ominus A	\ominus B	\ominus C	\ominus D
17	\ominus A	\ominus B	\ominus C	\ominus D
18	\ominus A	\ominus B	\ominus C	\ominus D
19	\ominus A	\ominus B	\ominus C	\ominus D
20	\ominus A	\ominus B	\ominus C	\ominus D
21	\ominus A	\ominus B	\ominus C	\ominus D
22	\ominus A	\ominus B	\ominus C	\ominus D
23	\ominus A	\ominus B	\ominus C	\ominus D
24	\ominus A	\ominus B	\ominus C	\ominus D
25	\ominus A	\ominus B	\ominus C	\ominus D
26	\ominus A	\ominus B	\ominus C	\ominus D
27	\ominus A	\ominus B	\ominus C	\ominus D
28	\ominus A	\ominus B	\ominus C	\ominus D
29	\ominus A	\ominus B	\ominus C	\ominus D
30	\ominus A	\ominus B	\ominus C	\ominus D
31	\ominus A	\ominus B	\ominus C	\ominus D
32	\ominus A	\ominus B	\ominus C	\ominus D
33	\ominus A	\ominus B	\ominus C	\ominus D
34	\ominus A	\ominus B	\ominus C	\ominus D
35	\ominus A	\ominus B	\ominus C	\ominus D
36	\ominus A	\ominus B	\ominus C	\ominus D
37	\ominus A	\ominus B	\ominus C	\ominus D
38	\ominus A	\ominus B	\ominus C	\ominus D
39	\ominus A	\ominus B	\ominus C	\ominus D
40	\ominus A	\ominus B	\ominus C	\ominus D
41	\ominus A	\ominus B	\ominus C	\ominus D
42	\ominus A	\ominus B	\ominus C	\ominus D
43	\ominus A	\ominus B	\ominus C	\ominus D
44	\ominus A	\ominus B	\ominus C	\ominus D
45	\ominus A	\ominus B	\ominus C	\ominus D
46	\ominus A	\ominus B	\ominus C	\ominus D
47	\ominus A	\ominus B	\ominus C	\ominus D
48	\ominus A	\ominus B	\ominus C	\ominus D
49	\ominus A	\ominus B	\ominus C	\ominus D
50	\ominus A	\ominus B	\ominus C	\ominus D
51	\ominus A	\ominus B	\ominus C	\ominus D
52	\ominus A	\ominus B	\ominus C	\ominus D
53	\ominus A	\ominus B	\ominus C	\ominus D
54	\ominus A	\ominus B	\ominus C	\ominus D
55	\ominus A	\ominus B	\ominus C	\ominus D
56	\ominus A	\ominus B	\ominus C	\ominus D
57	\ominus A	\ominus B	\ominus C	\ominus D
58	\ominus A	\ominus B	\ominus C	\ominus D
59	\ominus A	\ominus B	\ominus C	\ominus D
60	\ominus A	\ominus B	\ominus C	\ominus D
61	\ominus A	\ominus B	\ominus C	\ominus D
62	\ominus A	\ominus B	\ominus C	\ominus D
63	\ominus A	\ominus B	\ominus C	\ominus D
64	\ominus A	\ominus B	\ominus C	\ominus D
65	\ominus A	\ominus B	\ominus C	\ominus D
66	\ominus A	\ominus B	\ominus C	\ominus D
67	\ominus A	\ominus B	\ominus C	\ominus D
68	\ominus A	\ominus B	\ominus C	\ominus D
69	\ominus A	\ominus B	\ominus C	\ominus D
70	\ominus A	\ominus B	\ominus C	\ominus D
71	\ominus A	\ominus B	\ominus C	\ominus D
72	\ominus A	\ominus B	\ominus C	\ominus D
73	\ominus A	\ominus B	\ominus C	\ominus D
74	\ominus A	\ominus B	\ominus C	\ominus D
75	\ominus A	\ominus B	\ominus C	\ominus D
76	\ominus A	\ominus B	\ominus C	\ominus D
77	\ominus A	\ominus B	\ominus C	\ominus D
78	\ominus A	\ominus B	\ominus C	\ominus D
79	\ominus A	\ominus B	\ominus C	\ominus D
80	\ominus A	\ominus B	\ominus C	\ominus D
81	\ominus A	\ominus B	\ominus C	\ominus D
82	\ominus A	\ominus B	\ominus C	\ominus D
83	\ominus A	\ominus B	\ominus C	\ominus D
84	\ominus A	\ominus B	\ominus C	\ominus D
85	\ominus A	\ominus B	\ominus C	\ominus D
86	\ominus A	\ominus B	\ominus C	\ominus D
87	\ominus A	\ominus B	\ominus C	\ominus D
88	\ominus A	\ominus B	\ominus C	\ominus D
89	\ominus A	\ominus B	\ominus C	\ominus D
90	\ominus A	\ominus B	\ominus C	\ominus D
91	\ominus A	\ominus B	\ominus C	\ominus D
92	\ominus A	\ominus B	\ominus C	\ominus D
93	\ominus A	\ominus B	\ominus C	\ominus D
94	\ominus A	\ominus B	\ominus C	\ominus D
95	\ominus A	\ominus B	\ominus C	\ominus D
96	\ominus A	\ominus B	\ominus C	\ominus D
97	\ominus A	\ominus B	\ominus C	\ominus D
98	\ominus A	\ominus B	\ominus C	\ominus D
99	\ominus A	\ominus B	\ominus C	\ominus D
100	\ominus A	\ominus B	\ominus C	\ominus D

NAME: KEY

SUBJECT: Calculus

DATE: 2007

PERIOD: 200 (First)

John

ACCU-SCAN ADVANTAGE™ #2804C-FR 03/03 99076
 APPERSON PRINT MANAGEMENT SERVICES (800) 827-9219
 U.S. Patent No. 6,079,624 09/05